Finite-Difference Investigation of Axisymmetric Inviscid Separated Flows with Infinitely-Long Cusp-Ended Stagnation Zone. Flow around a Sphere

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Abstract

The classical Helmholtz problem is applied for modelling the axisymmetric inviscid cusp-ended separated flow around a sphere. Two coordinate systems are employed: polar for initial calculations and parabolic the latter being more suitable for investigation of infinitely long stagnation zones. Scaled coordinates are introduced and difference schemes for the free-stream equation and the Bernoulli integral are devised. The separation point is not initially prescribed and is defined iteratively. A separated flow with vanishing drag coefficient is obtained.

1. Introduction

In an attempt to explain the existence of a sizable drag force upon a submerged body even for vanishing viscosity, Helmholtz [10] introduced the notion of discontinuous ideal flow consisting of a potential and stagnant parts; these matching at an unknown stream surface. The idea of discontinuous ideal flow was successfully applied by Kirchhoff [11] for bodies with sharp edges and later developed by Levi-Civita [12], Villat [20], Brodetsky [4], etc. for bodies with curved profile when additional condition for smooth separation (Brillouin-Villat condition) is to be satisfied. All these solutions are planar and based on the hodograph method. Unfortunately this powerful tool is not capable for ideal flows characterized by axial symmetry. Therefore the efforts in solving of such kind flows is mainly confined to the numerical approach. Hitherto there are known several approximate methods for study of axisymmetric ideal flows. The most important methods appear to be: the integral one used at first by Trefftz [19] and later extended by Struck [16]; the relaxation one applied by Southwell&Vaisey [15] and developed by Brennen [3] (for detailed reference see [21, 9]). Similarly to the plane flows in the case of curve bodies a smooth separation condition or any else semi-empirical assumptions are suggested in order to yield satisfactory forecast concerning the velocity and pressure distribution, detachment point and drag coefficient [1, 2, 13]. Particularly Southwell&Vaisey by working in the physical plane obtained only cusp-ended cavity behind a sphere. We also calculated such kind stagnation zone behind a sphere [17] by means of finite-difference scheme at that

without pre-conditioning the separation point. Now we aim at utilizing the improved difference scheme, which was developed and applied for the planar inviscid flow around circular cylinder [18] for investigation of a separated axisymmetric flow around a sphere.

Following our approach we use two different coordinate systems: a polar spherical coordinate system for initial calculations and a parabolic coordinate system the latter being topologically more suited for solving the free-stream equation outside infinitely-long stagnation zones. We switch from polar coordinates to parabolic ones after the stagnation zone has fairly well developed and has become long enough.

2. Posing the Problem

Consider the steady inviscid flow past a circle – an arbitrary meridian cross section of a sphere. The direction of the flow coincides with the line $\theta = 0, \pi$ of the polar coordinates and the leading stagnation point of the flow is situated in the point $\theta = \pi$. The axially symmetry enables to study the flow in the meridian halfplane only.

Dimensionless variables are introduced as follows

$$\psi' = \frac{\psi}{L^2 U_\infty}, \quad r' = \frac{r}{L}, \quad q = \frac{p - p_c}{\frac{1}{2}\rho U_\infty^2}, \quad \sigma = \sqrt{L}\sigma', \quad \tau = \sqrt{L}\tau', \quad \kappa = \frac{p_\infty - p_c}{\frac{1}{2}\rho U_\infty^2}, \tag{2.1}$$

where L is the characteristic length of the body ($2a$ for a sphere of radius a), U_∞ – velocity of the undisturbed flow; p_c – the pressure inside the stagnation zone; p_∞ – the pressure at infinity, r - the polar radius, σ, τ-the parabolic coordinates, κ - the cavitation number, which for flows with stagnation zones is equal to zero. Without fear of confusion the primes will be omitted henceforth.

2.1. Coordinate Systems

In terms of the two coordinate systems (polar spherical and parabolic) equation for the stream function ψ reads:

$$\frac{1}{\sin\theta}(\psi_r)_r + \frac{1}{r^2}\left(\frac{\psi_\theta}{\sin\theta}\right)_\theta = 0\,, \qquad \text{or} \qquad \frac{1}{\tau}\left(\psi_\sigma + \frac{\psi}{\sigma}\right)_\sigma + \frac{1}{\sigma}\left(\psi_\tau + \frac{\psi}{\tau}\right)_\tau = 0\,. \tag{2.2}$$

The undisturbed uniform flow at infinity is given by

$$\psi|_{r\to\infty} \approx \frac{r^2 U_\infty \sin^2\theta}{2}\,, \qquad \text{or} \qquad \psi|_{\sigma\to\infty,\,\tau\to\infty} \approx \sigma\tau U_\infty\,. \tag{2.3}$$

On the combined surface “body+stagnation zone” hold two conditions. The first condition secures that the said boundary is a streamline (say of number “zero”)

$$\psi(R(\theta),\theta) = 0,\ \theta \in [0,\pi] \quad \text{or} \quad \psi(S(\tau),\tau) = 0,\ \tau \in (0,\infty)\,, \tag{2.4}$$

where $R(\theta)$, $S(\tau)$ are the shape functions of the total boundary in spherical or parabolic coordinates, respectively. As usually we use the notation Γ_1 for the portion of boundary representing the rigid body (the sphere) and Γ_2 – for the free streamline (Fig.1).

On Γ_2 the shape function $R(\theta)$ is unknown and it is to be implicitly identified from Bernoulli integral with the pressure equal to a constant (say, p_c) which is the second

condition holding on the free boundary. For the two coordinate systems one gets the following equations for shape functions $R(\theta)$ or $S(\tau)$:

$$\left[q + \frac{1}{r^2\sin^2\theta}(\frac{\psi_\theta^2}{r^2} + \psi_r^2)\right]_{r=R(\theta)} = 1\,, \qquad \text{or} \qquad \left[q + \frac{\psi_\sigma^2 + \psi_\tau^2}{\sigma^2 + \tau^2}\right]_{\sigma=S(\tau)} = 1\,. \tag{2.5}$$
$$\theta \in \Gamma_2 \qquad\qquad\qquad\qquad \tau \in \Gamma_2$$

The boundary value problem (2.2), (2.3), (2.4), (2.5) is completed with the additional symmetry conditions

$$\frac{\partial\psi}{\partial\theta} = 0\,,\ \theta = 0, \pi \qquad \text{or} \qquad \frac{\partial\psi}{\partial\tau} = 0\,,\ \tau = 0\,. \tag{2.6}$$

In spherical coordinates along with ψ it is convenient to introduce new function $\Psi = \frac{\psi}{r\sin\theta}$. Then the dynamical condition (2.5a) takes the form:

$$\left[q + \frac{\Psi_\theta^2}{r^2} + \Psi_r^2\right]_{r=R(\theta)} = 1\,. \tag{2.7}$$
$$\theta \in \Gamma_2$$

Obviously $\Psi_r|_{r=R(\theta)} = \left.\frac{\psi_r}{r\sin\theta}\right|_{r=R(\theta)}$, $\Psi_\theta|_{r=R(\theta)} = \left.\frac{\psi_\theta}{r\sin\theta}\right|_{r=R(\theta)}$. Without confusion we will name Ψ stream function too.

2.2. Scaled Variables

Following [6, 7, 8] we introduce new scaled coordinates:

$$\eta = rR^{-1}(\theta)\,, \qquad \eta = \sigma - S(\tau),$$

which render the original regions to semi-infinite strips.

If we denote $\xi \equiv \theta$ or $\xi \equiv \tau$ depending on the particular case under consideration, then in terms of the new coordinates (η, ξ), the governing equation (2.2) takes the form

$$A(\psi_\eta)_\eta + B(b\psi_\xi)_\xi - C(\psi_\xi)_\eta - D(d\psi_\eta)_\xi + (e\psi)_\eta + (f\psi)_\xi = 0\,, \tag{2.8}$$

where

$$b \equiv \frac{1}{\sin\theta}\,, \quad d \equiv \frac{R'}{R}\frac{1}{\sin\theta}\,, \quad e \equiv 0\,, \quad f \equiv 0;$$
$$A \equiv \eta^2 + \eta\left(\frac{R'}{R}\right)^2, \quad B \equiv \sin\theta\,, \quad C \equiv \eta\frac{R'}{R}\,, \quad D \equiv \eta\sin\theta\,;$$

or

$$b \equiv 1\,, \quad d \equiv S'\,, \quad e \equiv \frac{1}{\eta + S} - \frac{S'}{\tau}\,, \quad f \equiv \frac{1}{\tau};$$
$$A \equiv 1 + S'^2\,, \quad B \equiv 1\,, \quad C \equiv S'\,, \quad D \equiv 1\,.$$

Similarly to [18] we use the "relative" function $\bar{\psi}$

$$\bar{\psi}(\eta,\theta) = \psi(\eta,\theta) - \frac{[\eta R(\theta)\sin\theta]^2}{2}\,, \quad \bar{\psi}(\eta,\tau) = \psi(\eta,\tau) - (\eta + S(\tau))\tau\,,$$

which is obviously a solution to eq.(2.8) and which we loosely call stream function. The asymptotic boundary condition then becomes

$$\bar{\psi}\Big|_{\eta=\eta_\infty} = 0 \qquad \text{or} \qquad \bar{\psi}\Big|_{\eta=\eta_\infty,\,\tau=\tau_\infty} = 0\,, \tag{2.9}$$

while the non-flux condition on Γ transforms as follows

$$\bar{\psi}\Big|_{\eta=1} = -\frac{[R(\theta)\sin\theta]^2}{2} \qquad \text{or} \qquad \bar{\psi}\Big|_{\eta=0} = -S(\tau)\tau\,. \tag{2.10}$$

Thus eqs.(2.8), (2.9), (2.10), (2.6) define a well posed boundary value problem provided that functions $R(\theta)$ and $S(\tau)$ are known. On the other hand in the portion Γ_2 of the boundary (where these functions are unknown) they can be evaluated from the Bernoulli integral (2.5) and (2.7) which now becomes an explicit equation for the shape function

$$\begin{aligned} \frac{R^2+R'^2}{R^6\sin^2\theta}\left[\frac{\partial\bar{\psi}}{\partial\eta}\Big|_{\eta=1} + R^2(\theta)\sin^2\theta\right]^2 &= 1 \\ & \qquad 0 \le \theta \le \theta^* \\ \frac{R^2+R'^2}{R^4}\left[\frac{\partial\bar{\Psi}}{\partial\eta}\Big|_{\eta=1} + R(\theta)\sin\theta\right]^2 &= 1\,, \\ & \text{or} \\ \frac{1+S'^2}{S^2+\tau^2}\left[\frac{\partial\bar{\psi}}{\partial\eta}\Big|_{\eta=0} + \tau\right]^2 &= 1\,, \quad \tau^* \le \tau < \infty\,. \end{aligned} \tag{2.11}$$

Here $\bar{\Psi}(\eta,\theta) = \Psi(\eta,\theta) - \eta R(\theta)\sin\theta$.

3. Forces Exerted on the Body

Apparently the presence of a stagnation zone breaks the symmetry of the integral for the normal stresses and hence D'Alembert paradox is not hold. If denote by $\boldsymbol{n}$ the outward normal vector to the sphere Σ and by $d\sigma$ - the surface element of the sphere, then the force acting upon the body is given by

$$\boldsymbol{R} = -\oint_\Sigma p\boldsymbol{n}d\sigma \tag{3.1}$$

It is not difficult to obtain for the drag and lifting-force coefficients of every meridian cross section the following expressions

$$C_x = -\int_{\theta^*}^{\pi} qR(\theta)\sin(\theta)\,[R(\theta)\cos\theta + R'(\theta)\sin\theta]\,d\theta \quad \text{or} \quad C_x = \int_0^{\tau^*} qS(\tau)\tau\,[S(\tau) + S'(\tau)\tau]\,d\tau$$
$$C_y \;\equiv\; 0\,, \tag{3.2}$$

where the dimensionless pressure is given by

$$
\begin{aligned}
q &= 1-\frac{R^2+R'^2}{R^4}\left[\left.\frac{\partial\bar{\Psi}}{\partial\eta}\right|_{\eta=1}+R\sin\theta\right]^2 \\
&\text{or} \\
q &= 1-\frac{1+S'^2}{S^2+\tau^2}\left[\left.\frac{\partial\bar{\psi}}{\partial\eta}\right|_{\eta=0}+\tau\right]^2 .
\end{aligned}
\tag{3.3}
$$

4. Difference Scheme and Algorithm

4.1. Splitting scheme for the free-stream equation

The computational domain being infinite is reduced to finite one after appropriately choosing the "actual infinities". In order to take into consideration the topological and dynamic features of the flow we employ non-uniform mesh, which was presented in detail at [18].

Let us denote the spacings of the mesh by $h_{i+1} \equiv \eta_{i+1}-\eta_i$, $i = 1,\dots,M$ and $g_{j+1} \equiv \xi_{j+1}-\xi_j$, $j = 1,\dots,N$. We solve the boundary value problem iteratively using the method of splitting of operator. Upon introducing fictitious time we render the equation to parabolic type and then employ the so-called scheme of stabilising correction [22]. On the first half-time step we have the following differential equations (Δt is the time increment)

$$
\begin{aligned}
\frac{\psi_{ij}^{n+\frac{1}{2}}-\psi_{ij}^{n}}{\frac{1}{2}\Delta t} &= B_{ij}\Lambda_2(b\Lambda_2\psi^{n+\frac{1}{2}})_{ij}+A_{ij}\Lambda_1(\Lambda_1\psi^n)_{ij}-C_{ij}\Lambda_1(\Lambda_2\psi^n)_{ij} \\
&\quad -D_{ij}\Lambda_2(d\Lambda_1\psi^n)_{ij}+\Lambda_1(e\psi^n)_{ij}+\Lambda_2(f\psi^n)_{ij}
\end{aligned}
\tag{4.1}
$$

for $i = 2,\cdots,M,\ j = 2,\dots,N$

The second half-time step consists in solving the following differential equations

$$
\frac{\psi_{ij}^{n+1}-\psi_{ij}^{n+\frac{1}{2}}}{\frac{1}{2}\Delta t} = A_{ij}\left(\Lambda_1(\Lambda_1\psi^{n+1})_{ij}-\Lambda_1(\Lambda_1\psi^n)_{ij}\right)
\tag{4.2}
$$

for $i = 2,\dots,M,\ j = 2,\dots,N$. The last two equations (4.1)-(4.2) are completed with respective boundary conditions [7]. Here

$$
\Lambda_1(.)_{ij} \equiv \frac{\partial}{\partial\eta}(.)_{ij}+O(h_ih_{i+1}) ,
$$

$$
\Lambda_2(.)_{ij} \equiv \frac{\partial}{\partial\xi}(.)_{ij}+O(g_jg_{j+1})
$$

are the usual difference operators based on three-point patterns with second order of approximation.

Thus the b.v.p. for the stream function is reduced to consequative systems with sparse (three-diagonal) matrices, which are solved iteratively [7].

Since the condition for numerical stability of the elimination is not satisfied for all points of domain here a "non-monotonous progonka" (see [14, 5]) is employed like at [18].

4.2. Difference Approximation for the Free Boundary

Following [18] in the present work we use the dynamic condition (2.5) in spherical coordinates only, so that we present here just the relevant scheme in spherical coordinates. The equations (2.11) can be resolved for the derivative $R'(\theta)$ when the following conditions are satisfied:

$$Q(\theta) \stackrel{\text{def}}{=} \frac{R^4(\theta)\sin^2\theta}{T^2} > 1\ ,\ T = \left.\frac{\partial\bar{\psi}}{\partial\eta}\right|_{\eta=1} + (R(\theta)\sin\theta)^2$$

or (4.3)

$$Q(\theta) \stackrel{\text{def}}{=} \frac{R^2(\theta)}{\mathcal{T}^2} > 1\ ,\ \mathcal{T} = \left.\frac{\partial\bar{\Psi}}{\partial\eta}\right|_{\eta=1} + R(\theta)\sin\theta\ .$$

The above inequalities are trivially satisfied in the vicinity of the rear-end stagnation point inasmuch as that for $\theta \to 0$ one has $T \to 0$ or $\mathcal{T} \to 0$ and hence $\frac{R^4\sin^2\theta}{T^2} \to \infty$ or $\frac{R^2}{\mathcal{T}^2} \to \infty$. The first inequality, however, is indeterminated at the point $\theta = 0$ due to the ratio $\frac{\sin\theta}{T(\theta)}$.

For the shape function $\hat{R}_j$ of free line is solved the following difference scheme

$$\hat{R}_{j-1} - \hat{R}_j = g_j \frac{\hat{R}_j + \hat{R}_{j-1}}{2}\sqrt{\frac{1}{2}\left[\left(\frac{(R_j^\alpha)^2\sin\theta_j}{T_j^\alpha}\right)^2 + \left(\frac{(R_{j-1}^\alpha)^2\sin\theta_{j-1}}{T_{j-1}^\alpha}\right)^2\right] - 1}$$

or (4.4)

$$\hat{R}_{j-1} - \hat{R}_j = g_j \frac{\hat{R}_j + \hat{R}_{j-1}}{2}\sqrt{\frac{1}{2}\left[\left(\frac{R_j^\alpha}{\mathcal{T}_j^\alpha}\right)^2 + \left(\frac{R_{j-1}^\alpha}{\mathcal{T}_{j-1}^\alpha}\right)^2\right] - 1}$$

for $j = j^*, \ldots, 2$, whose approximation is $O(g_j^2)$. Only in the detachment point the difference scheme is different, specifying in fact the initial condition, namely

$$\hat{R}_{j^*} - R(\theta^*) = g^* \frac{R(\theta^*) + \hat{R}_{j^*}}{2}\sqrt{\frac{1}{2}\left[\left(\frac{(R_{j^*}^\alpha)^2\sin\theta_{j^*}}{T_{j^*}^\alpha}\right)^2 + \left(\frac{R^2(\theta^*)\sin\theta^*}{T(\theta^*)}\right)^2\right] - 1}$$

or

$$\hat{R}_{j^*} - R(\theta^*) = g^* \frac{R(\theta^*) + \hat{R}_{j^*}}{2}\sqrt{\frac{1}{2}\left[\left(\frac{R_{j^*}^\alpha}{\mathcal{T}_{j^*}^\alpha}\right)^2 + \left(\frac{R(\theta^*)}{\mathcal{T}(\theta^*)}\right)^2\right] - 1}\ ,$$

where R without a superscript or "hat" stands for the known boundary of rigid body.

At last a relaxation is used for the shape-function of the free boundary at each global iteration α according to the formula:

$$R^{\alpha+1} = \omega\hat{R}_j + (1-\omega)R_j^\alpha\ ,$$

where ω is called relaxation parameter.

4.3. The general Consequence of the Algorithm

Each global iteration contains two stages. On the first stage, the difference problem for free-stream equation is solved iteratively either in polar spherical or in parabolic coordinates (depending on the development of the stagnation zone).

The second stage of a global iteration consists in solving the difference problem for the free surface in polar spherical coordinates.

Through the indetermination at the axis of symmetry we use the difference scheme (4.4a) only during the first several iterations (in polar spherical coordinates). The calculation of the shape of the far weak (in parabolic coordinates) we carry out using the scheme (4.4b). The latter appears to be more convenient and efficient because the loss of accuracy and 'numerical' instability in vicinity of the axis of cusp are avoided. The criterion for convergence of the global iterations is defined by the convergence of the shape function, namely

$$\max_j \left| \frac{R_j^{\alpha+1} - R_j^{\alpha}}{R_j^{\alpha+1}} \right| < 10^{-4}. \tag{4.5}$$

The obtained solutions for the stream function and the shape function of the boundary are the values of the last iteration $\psi_{ij} = \psi_{ij}^{\alpha+1}$ and $R_j = R_j^{\alpha+1}$, respectively. Then the velocity, pressure, and the forces exerted from the flow upon the body are calculated.

5. Results and Discussion

The numerical correctness of scheme (4.1), (4.2) is verified through usual experiments including a doubling the mesh knots and varying the 'actual infinity' We used different meshes with sizes M x N : 41x68, 81x136, 101x202, etc. Respectively, the actual infinity η_∞ assumed in the numerical experiments the values 10, 20. The dependence of the numerical solution on the time increment Δt is also investigated and it is shown that the scheme of fractional steps for the stream function has a full approximation [22]. Comparing the different finite-difference realizations of the solution we choice the following 'optimal' values of the governing parameters: step of the fictitious time $\Delta t = 0.5$, relaxation $\omega = 0.01$ and 'actual' infinity $\eta_\infty = 10$.

In Fig.2-a are presented the obtained shapes of the stagnation zone behind the sphere and in the near wake for resolutions 41×68, 81×136 and 101×202 and value of relaxation parameter: $\omega = 0.01$.

Evidently the agreement among the calculated shapes of the free boundary near the body corresponding to these three meshes is very well. The logarithmic scale is used in Fig.2-b in order to expand the differences between the different solutions making them visible in the graph. As clearly it is shown the curves are indistinguishable till distance 150 calibers and the relative error is less than 1%. The relative error between the meshes 81×136 and 101×202 at distances more than 150 calibers does not exceed 4%. At the same time the relative error between the mesh 41×68 and the else two ones increases and reaches 7-8% at distance 200 calibers. Obviously that mesh is not enough fine and appears to be coarse for calculating the shape function at large distances behind the sphere. The obtained results warrant conclusion that the scheme is fully effective in solving the free boundary till 200 calibers. The very good comparison supports the claim that indeed a solution to the Helmholtz problem has been found numerically by means of the developed in the present work difference scheme. The calculated here dimensionless pressure q is

shown in Fig.3. The agreement among the obtained pressure curves corresponding to different mesh resolutions is excellent. In the stagnation zone the pressure is in order of 10^{-4} in accordance with the assumption that the unknown boundary is defined by the condition $q = 0$. The amplitude of the minimum of q is smaller than 1.25 the latter being the value for ideal flow without separation. This means that the stagnation zone influences the flow upstream. The calculated magnitude of the separation angle (measured with respect the rear end of the sphere) varies between $69.42°$ for mesh 41×68 and $69.7°$ for mesh 101×202. It is interesting to note that the calculated here drag coefficient C_x has a magnitude between $.5848 \times 10^{-3} - .5704 \times 10^{-2}$ obtained for the different resolutions, i.e., we conclude that in order of approximation of the scheme $C_x = 0$. Then similarly to the separated flow around circular cylinder we can name the obtained separation angle 'critical' (see [18, 9]). Hence in the case of axisymmetric flow around sphere there also exists a inviscid separated flow for which the D'Alembert paradox holds. Trough the disscused features of the obtained Helmholtz flow we can assume it is an axisymmetric analogue of the Chaplygin-Kolscher flow around circular cylinder.

6. Concluding Remarks

The separated inviscid flow behind a sphere is treated as a flow with free surface – the boundary of the stagnation zone (Helmholtz problem). Scaled coordinates are employed rendering the computational domain into a region with fixed boundaries and transforming the Bernoulli integral into an explicit equation for the shape function. A new free-stream function is introduced and thus the numerical instability near the symmetry axis is avoided. Difference scheme using coordinate splitting is devised. Exhaustive set of numerical experiments is run and the optimal values of scheme parameters are defined. Results are verified on grids with different resolutions. The obtained here shape of the stagnation zone is of infinitely long cusp and respective separated flow has vanishing drag coefficient. The detachment point is not prescribed in advance and it is defined iteratively satisfying the mere Bernoulli integral there.

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FIGURE CAPTIONS

fig1.gif

Figure 1: Posing the problem

sphnear.gif

(a) behind the sphere

sphfar.gif

(b) far wake

Figure 2: The obtained separation lines for relaxation parameter $\omega = 0.01$ and different resolutions: - - - - 41×68; — — — 81×136; − − − 101×202.

sphpres.gif

Figure 3: The pressure distribution for relaxation parameter $\omega = 0.01$ and different resolutions: - - - - 41×68; — — — 81×136; − − − 101×202; —— inviscid nonseparated flow.